Comparison of SPH and FEM in approach Thermomechanical Coupled Problems.

Krzysztof Damaziak, [1, a)] and Jerzy Małachowski, [1, b)]

(Use the Microsoft Word template style: *Paper Author*)

[1]Department of Mechanics and Applied Computer Science
Faculty of Mechanical Engineering, Military University of Technology
00-908 Warsaw, Witolda Urbanowicza Street 2, Poland

[a)]Corresponding author: krzysztof.damaziak@wat.edu.pl
[b)]jerzy.malachowski@wat.edu.pl

Abstract. After almost 40 years of development, meshless methods are still treated as "new" numerical technology. Smoothed Particles Hydrodynamics (SPH) is not an exception. Number of publications shows still new applications of the method. One of such rather new area is analysis of coupled thermal – mechanical field. In the article, authors show results of comparative analysis of SPH and FEM aimed to analyze usefulness of meshless approach in analysis of disk brake and lining interaction during braking. Several different tests allow to conclude on results obtained by meshless method and classic Finite Element Method in various scenarios of thermal – mechanical coupling.

DIFFERENCES BETWEEN SPH AND FEM

The principle of both methods can be found in number of references. Detailed description of SPH is shown in the (Liu and Gu, 2005) or in article by Liu (Liu and Liu, 2010), to mention the most popular sources. Actually, most of the articles dealing with SPH starts with more or less detailed explanation of theory lying behind the method. FEM, as more mature and more popular engineering tool, has even richer bibliography. Starting with classic work of Zienkiewicz (Zienkiewicz, 1977), one can also mention detailed lecture by Bathe (Bathe, 1996) and many, many other.

In the presence of huge amount of sources, authors decided to not include yet another derivation of meshless and FE methods fundamentals. Instead, a few consequences of the mathematical apparatus will be stressed out. In the presented examples, thermal part is shrink to a conduction problem, described by the Fourier's equation:

$$\frac{dE_T}{dt} = \frac{1}{\rho}\nabla\left(k\nabla\Theta\right) \tag{1}$$

where: E_T – thermal energy per unit mass ρ – density, Θ – temperature, k – thermal conductivity.

This applies for both methods, meaning that the equation (1) is the governing equation in FEM and in SPH. In case of FEM, above equation takes form (Lewis, Nithiarasu and Seetharamu, 2004):

$$\mathbf{C}\frac{d\mathbf{\Theta}}{dt} + \mathbf{K\Theta} = \mathbf{R}\ , \tag{2}$$

where: **C** – heat capacity matrix, **K** – conductivity matrix, **R** – vector of hat sources, while in SPH equation (1) is approximated by (Xu, 2012):

$$\frac{dE_T}{dt} = \sum_j \frac{m_j}{\rho_i\rho_j}\frac{k_i - k_j}{\left|x_{ij}\right|^2} x_{ij}\nabla_i W_{ij}\ \ , \tag{3}$$

where: m – mass of particle, W – kernel function x – location of particles.

VERIFICATION TEST

One of the four benchmarks was aimed to verification of the algorithms responsible to conversion of work done plastic deformation into heat. Simplified simulation of Taylor test was chosen as an physical basis of the numerical test. The numerical model is presented in Figure 1. Impactor was modeled by FE and SPH respectively. Its mechanical properties were described using bilinear material model.

At first, numerical model was built so that the number of particles was the same as number of nodes. First analyses showed, that SPH exhibits tension instabilities. The problem was solved by increase number of particles by 4 (double the number in two directions).

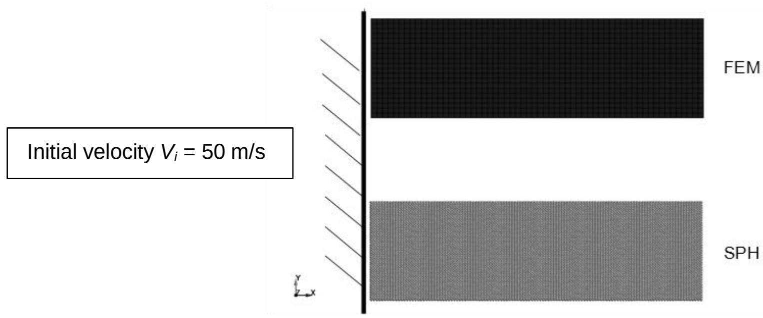

FIGURE 1. FEM and SPH domains in benchmark inspired by Taylor test

Distribution of the temperature generated by work done by plastic deformation is showed in Figure 2.

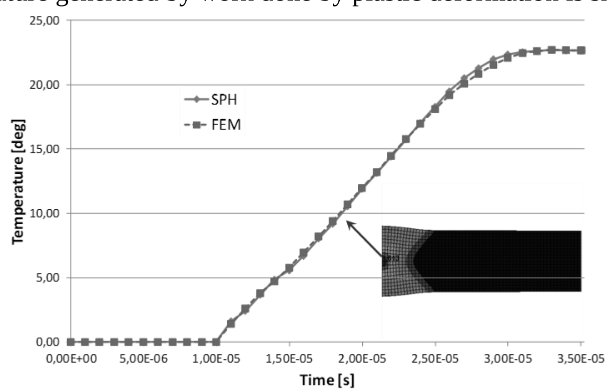

FIGURE 2 Temperature vs. time in reference point

Obtained results shows, that despite different mathematical description and even usage of different governing equations, properly used numerical methods leads to the same results.

REFERENCES

1. Bathe K-J., *Finite Element Procedures*, Prentice Hall, New Jersey (1996), ISBN: 0-13-301458-4
2. Liu G.R., Gu Y.T., 2005, *An Introduction to meshfree methods and their programming*, Springer, Dordrecht (2005), ISBN-10: 9048168198
3. Liu M.B., Liu G.R., Smoothed Particle Hydrodynamics (SPH): an Overview and Recent Developments, *Arch. Comput. Methods Eng*. 17, 25-76 (2010)
4. Zienkiewicz O.C., *The Finite Element Method- 3rd Edition*, McGraw-Hill (1977),
5. Xu J., Heat Transfer with Explicit SPH Method in LS-Dyna, *Proc. of 12th Int. LS-Dyna Users Conference* (2012)